

APPENDIX 1 – Trade-off between a coopetitive project and private activities

Assume that in a preliminary stage, each firm i decides how much of its budget ω_i to allocate to the coopetive project (n_i) and how much to allocate to a private productive activity ($\omega_i - n_i$). Let $\Phi_i(\omega_i - n_i)$ be the strictly increasing concave ($\Phi_i'(\cdot) > 0$ and $\Phi_i''(\cdot) < 0$) profit function of the private activity and $\Pi_i(\omega_i, n_i, \alpha_i)$ be the total profit that results from the firm's allocation decision.

The total profit $\Pi_i(\omega_i, n_i, \alpha_i)$ is given by

$$\Pi_i(\omega_i, n_i, \alpha_i) = \Phi_i(\omega_i - n_i) + M_i \cdot \sum_{j=1}^{j=K} \alpha_j n_j \cdot \frac{(1 - \alpha_i)n_i}{\sum_{j=1}^{j=K} (1 - \alpha_j)n_j}.$$

Given the optimal profit of the coopetitive project (Equation 7 in the paper), we have

$$\Pi_i(\omega_i, n_i, \alpha_i) = \Phi_i(\omega_i - n_i) + \pi_i(\alpha_i^*, \alpha_{-i}^*)$$

In looking for n_i^* , the FOC yields

$$\frac{\partial \Pi_i(\omega_i, n_i, \alpha_i)}{\partial n_i} = -\Phi_i'(\omega_i - n_i) + \frac{\partial \pi_i^*}{\partial n_i} = -\Phi_i'(\omega_i - n_i) + \frac{M_i}{K^2} = 0.$$

With Equations 6 and 8, we obtain

$$\Phi_i'(\omega_i - n_i) = \frac{M_i}{K^2}$$

$$n_i^* = \omega_i - \Phi_i'^{-1}\left(\frac{M_i}{K^2}\right).$$

The latter expression shows that the optimum budget dedicated by firm i to the coopetitive project is a function of the exogenous parameters: the number of firms (K), firm i 's exogenous appropriation capacity (M_i) and its total endowment (ω_i). Any change in those parameters directly affects the optimum dedicated budget:

- (i) firms with larger endowments contribute more to the coopetitive agreement, noting that an increase in ω_i increases n_i^* by the same amount ($\frac{\partial n_i^*}{\partial \omega_i} = 1$);
- (ii) an increase of the exogenous appropriation capacity (M_i) has a positive impact on n_i^* ;
- (iii) an increase in the number of firms (K) has a negative effect on n_i^* .

Property (i) is because the marginal profit of the private activity $\Phi_i'(\cdot)$ depends only on the number of firms and the exogenous appropriation capacity, and not on the available budget. Therefore, any additional budget at the optimum level will be dedicated to the coopetitive project.

APPENDIX 2 – Proof of the Nash equilibrium

Maximizing $\pi_i(\alpha_i, \alpha_{-i})$ according to α_i is equivalent to maximizing $\pi_i(\beta_i, \beta_{-i})$ with $\beta_i = 1 - \alpha_i$

$$\begin{aligned} \max_{\alpha_i} \pi_i(\alpha_i, \alpha_{-i}) &= M_i \cdot \sum_{j=1}^{j=K} \alpha_j n_j \cdot \frac{(1 - \alpha_i) n_i}{\sum_{j=1}^{j=K} (1 - \alpha_j) n_j} \\ \Leftrightarrow \max_{\beta_i} \pi_i(\beta_i, \beta_{-i}) &= M_i \cdot \sum_{l=1}^{l=K} (1 - \beta_l) n_l \cdot \frac{\beta_i n_i}{\sum_{j=1}^{j=K} \beta_j n_j} \text{ with } \beta_i = 1 - \alpha_i \end{aligned}$$

The FOC yields

$$\forall i \in [1, K], \quad n_i \cdot \frac{\sum_{j \neq i} \beta_j n_j}{(\sum_j \beta_j n_j)^2} \cdot \left(\sum_j (1 - \beta_j) n_j \right) - n_i \cdot \frac{\beta_i n_i}{\sum_j \beta_j n_j} = 0$$

Once simplified, we find

$$\begin{aligned} \forall i \in [1, K], \quad & \frac{\sum_{j \neq i} \beta_j n_j}{\sum_j \beta_j n_j} \cdot \left(\sum_j (1 - \beta_j) n_j \right) - n_i \beta_i = 0 \\ \forall i \in [1, K], \quad & \frac{\sum_{j \neq i} \beta_j n_j}{\sum_j \beta_j n_j} \cdot \left(\sum_j (1 - \beta_j) n_j \right) = n_i \beta_i \end{aligned}$$

Because this result holds for any participating firm, for example, firm k , and because the left-hand side of the equation is independent of i or k , we can conclude that

$$\forall i, k \in [1, K] \quad n_i \beta_i = n_k \beta_k$$

By substituting the other $n_j \beta_j$ with $n_i \beta_i$ in the FOC, we find

$$\begin{aligned} \forall i \in [1, K], \quad & \frac{(K-1)\beta_i n_i}{K\beta_i n_i} \cdot \left(\sum_j n_j - K\beta_i n_i \right) - \beta_i n_i = 0 \\ \forall i \in [1, K], \quad & \frac{(K-1)}{K\beta_i n_i} \cdot \left(\sum_j n_j - K\beta_i n_i \right) - 1 = 0 \\ \forall i \in [1, K], \quad & (K-1) \cdot \left(\sum_j n_j - K\beta_i n_i \right) = K\beta_i n_i \\ \forall i \in [1, K], \quad & (K-1) \sum_j n_j = K^2 \beta_i n_i \\ \forall i \in [1, K], \quad & \beta_i^* = \frac{(K-1) \sum_j n_j}{K^2 n_i} \end{aligned}$$

From the above, we can conclude that

$$\begin{aligned} \forall i \in [1, K], \quad & \alpha_i^* = 1 - \beta_i^* = 1 - \frac{(K-1) \sum_j n_j}{K^2 n_i} \\ \forall i \in [1, K], \quad & \alpha_i^* = \frac{K^2 n_i}{K^2 n_i} - \frac{(K-1) \sum_{j \neq i} n_j + (K-1) n_i}{K^2 n_i} \\ \forall i \in [1, K], \quad & \alpha_i^* = \frac{K^2 - (K-1)}{K^2} - \frac{K-1}{K^2} \cdot \frac{\sum_{j \neq i} n_j}{n_i} \end{aligned}$$

Biographical notes

Paul CHIAMBARETTO is an Associate Professor of Strategy and Marketing at Montpellier Business School and Associate Researcher at Ecole Polytechnique. His main research topics include inter-organizational relationships (such as alliances, alliance portfolios and coopetition). Over the years, he has developed a strong expertise in the air and rail transportation industries. His research has been published in ranked journals, such as *Research Policy*, *Long Range Planning*, *Industrial Marketing Management*, *International Studies of Management and Organization*, *M@n@gement*, etc. He is also one of the co-editors of the *Routledge Companion to Coopetition Strategies*. He has been a visiting researcher in several foreign institutions, such as the University of Oxford (UK), Concordia University (Canada) and Umea University (Sweden).

Jonathan MAURICE is an Associate Professor of Management Accounting at Toulouse School of Management, Toulouse Capitole University and member of TSM Research. His research focuses on management control systems and environmental accounting. He is particularly interested in the influences of management control systems on behaviors within and between organizations and how accountants address uncertainty regarding ‘environmental’ accounting numbers. He uses various research methods, such as case studies, interventionist research, meta-analyses, surveys and experiments. He recently published articles regarding the paradox of accounting innovation adoption, such as the ABC costing method, and the reliability of environmental accounting provisions. His ongoing projects investigate the influence of interactive management control tools on cooperation in a horizontal network of organizations and the implementation of an accounting and budgeting risk management system in the public sector.

Marc WILLINGER is a Professor of Economics at the University of Montpellier (UM) and member of the Center for Environmental Economics of Montpellier (CEE-M). His main research area is experimental and behavioral economics. His research mainly relies on laboratory and field experiments to study risk taking and social preferences in the context of social dilemma and asymmetric information. His current research interests include the local adaptation to a risky environment and trading behavior in experimental asset markets with a focus on socially responsible investments.